

## NDIR: Origin of Non-Linearity and SPAN

The dual channel detector in an NDIR sensor consists of two filter windows. The active filter is centred on an infrared absorption peak of the target molecule and the reference window is in a portion of the infrared spectrum where there are no absorption bands. Although the active filter is centred on an absorption band, there are wavelengths that cannot be absorbed due to the fine structure in the absorption spectra and slight mis-match between the filter bandwidth and the absorption band. The measured intensity of radiation at the active detector is therefore the sum of an absorbing intensity,  $I_a$ , and a passive intensity,  $I_p$ :

$$I = I_a + I_p$$

Assume the passive intensity is a fixed proportion of the incident radiation,  $I_0$ :

$$I_p = aI_0$$

The change in the active intensity with distance is given by:

$$\frac{dI_a}{dz} = -sNI_a$$

where  $dz$  is a section of infinitesimal length,  $s$  is the cross-section of the absorbing molecule and  $N$  is the number of molecules.

Since  $I = I_a + I_p$  and by definition  $I_p$  does not change:

$$\frac{dI}{dz} = \frac{dI_a}{dz} = \frac{d(I - aI_0)}{dz} = -sNI_a = -sN(I - aI_0)$$

Through separation of variables:

$$\frac{dI}{(I - aI_0)} = -sNdz$$

Integrating where  $k$  is the path length:

$$\int_{I_0}^I \frac{1}{I - aI_0} dI = \int_0^k -sNdz$$

$$\ln(I - aI_0) - \ln(I_0 - aI_0) = -sNk$$

If the path length,  $k$ , is fixed then  $N$  is proportional to the concentration,  $x$ , through a constant,  $b$ :

$$\ln\left(\frac{I - aI_0}{I_0(1 - a)}\right) = -bx$$

$$\left( \frac{I - aI_0}{I_0(1-a)} \right) = \exp(-bx)$$

$$\frac{I}{I_0} = a + (1-a)\exp(-bx)$$

Substituting  $a$ , which is the percentage of non-absorbing radiation for  $(1-d)$ , where  $d$  is the proportion of absorbing radiation:

$$\frac{I}{I_0} = (1-d) + d\exp(-bx)$$

$$\frac{I}{I_0} = 1 - d(1 - \exp(-bx))$$

If absorbance is defined as:

$$ABS = 1 - \left( \frac{I}{I_0} \right)$$

Then:

$$ABS = d(1 - \exp(-bx))$$

However, in practice it is found that this does not give a perfect fit (due to factors such as variations in path length and light scattering) and it is necessary to introduce a power term so that:

$$ABS = d(1 - \exp(-bx^c))$$

$d$  (the proportion of absorbing radiation) is also known as the *SPAN* and substitution yields the equation:

$$ABS = SPAN(1 - \exp(-bx^c))$$

which is used throughout these Application Notes.

The coefficients  $b$  and  $c$  are known as linearisation coefficients and are fixed for each concentration range (see Table 1 in AAN 201).